1. INTRODUCTION

Does logic—in particular, classical logic—have metaphysical presuppositions? It may be thought that it doesn’t: logical principles and logical inferences are often taken as not requiring the existence of any objects for them to hold. Logical principles are supposedly true in any domain (so there is no reliance on the subject matter at hand), and logical inferences are traditionally understood as being similarly independent of the subject matter under consideration. As Rudolf Carnap famously pointed out:

If logic is to be independent of empirical knowledge, then it must assume nothing concerning the existence of objects (Carnap [1937], p. 140; italics in the original).

In this paper I examine a number of arguments to the contrary, according to which, despite appearances, logical principles and logical inferences do have metaphysical presuppositions. I consider critically these arguments and indicate how they can be resisted, and motivate an alternative that, I argue, recognizes the nature and limitations of such presuppositions. In the end, as will become clear, it does require a suitable understanding of logic and some strategies to avoid commitment to abstract objects to block such presuppositions. Left unassisted, logic may not be presupposition free after all.

I should note that I am not using ‘presupposition’ in any technical sense. For the purposes of this paper, the presuppositions of a logical principle are those tacit assumptions that need to be true in order for the principle to hold. Also for the purposes of this paper, I am considering ‘logic’, and ‘logical principle’, very broadly. As will become clear, some of the considerations I will make are concerned with logical inferences, others with logical principles, and yet others address the (mathematical) framework that is used to develop a semantic analysis of the principles and inferences in question. All of them are part of ‘logic’ broadly understood.

2. ARGUMENTS FOR THE METAPHYSICAL PRESUPPOSITIONS OF LOGICAL PRINCIPLES AND LOGICAL INFERENCES

There are a number of arguments in support of the metaphysical presuppositions of logical principles and inferences. I will be unable, of course, to examine all of them. But I will examine seven that are particularly significant.

(a) Domains. Quantifiers range over domains, which in classical logic are typically taken to be non-empty sets. These domains are crucial to specify the range of the quantifiers, and without
them, it is not determined what the quantifiers ultimately cover. One cannot consistently claim that quantifiers range over absolutely everything, since this would include, for instance, totalities that are too large to be sets or inconsistent objects, such as the Russell set. If it is argued that such domains include absolutely everything that exists, and thus, no such inconsistent objects are in the range of the quantifiers, it becomes clear that domains provide more than simple scope delimitation. They exhibit what exists. But what exists is not a matter of logic, but a matter of metaphysics. Thus, domains of quantification are not so innocent or presupposition free.

Domains are also crucial in the usual model-theoretic characterization of logical consequence. On this characterization, domains of interpretation are, again, (non-empty) sets, and models are structures of a formal language composed by a domain \( D \) and an interpretation function that assigns objects in \( D \), relations or functions on \( D \) to the corresponding expressions of the language. In this case, \( B \) follows logically from \( A \) (in symbols, \( A \models B \)) as long as in every model in which \( A \) is true \( B \) is true as well. But if the domains of the models are sets, there are constraints on what can be considered as admissible domains. If no restrictions are imposed on the class of models, violations of principles of classical logic emerge. For example, if inconsistencies are not excluded (and thus for some \( A \), \( A \land \neg A \)), explosion—the rule according to which everything follows from a contradiction—is undermined. If incompleteness is admitted (and thus for some \( A \), it is not the case that \( A \) nor is it the case that \( \neg A \)), excluded middle is violated. Thus, for the model-theoretic conception to work in a classical setting, domains need to satisfy very definite conditions. Otherwise, classical logic is lost. But what is the justification to exclude certain objects from the domain? As will become clear, in each of these cases, metaphysical assumptions are in place.

(b) Identity. In classical logic, quantification presupposes the identity of the objects that are quantified over. Consider, for example, the inference from an arbitrary \( a \) that is \( F \) to every \( a \) is \( F \); that is, assuming that \( a \) is arbitrary and does not occur free in \( F \), \( Fa \models \forall x \, Fx \). This inference expresses the fact that if each object in the domain of quantification is \( F \), then every object in the domain is \( F \). But this is only the case if each distinct object in the domain is in the range of the universal quantifier. After all, if the same object were quantified over again and again and in each case it were \( F \), clearly this would not support the conclusion that every object is \( F \). This suggests that, at least in classical logic, quantification requires identity of the objects that are quantified over.

An additional way of making this point is by noting that it is a theorem of first-order classical logic with identity that \( \forall x \, x = x \). Of course, in this case, the identity of the objects that are quantified over is clearly presupposed. But is this a logical presupposition or rather a metaphysical one? It may be thought that every object’s self-identity is a fact about logic alone. However, the very idea that identity applies to every object in the domain has been challenged in some interpretations of non-relativist quantum mechanics, according to which identity is not defined for quantum particles (for a discussion, see French and Krause [2006]). If identity were a logical notion, its extension would be defined in every interpretation of one’s language. But arguably this is not the case in the context of these interpretations of quantum mechanics. Rather than a matter of logic, we seem to have here an additional metaphysical presupposition (namely, that identity applies to every object in the relevant domain).

(Note that it is not claimed that some quantum particles are not identical to themselves, which doesn’t seem to be intelligible. Rather the point is that, on the relevant interpretations of non-
relativist quantum mechanics, identity is not defined for these particles (see French and Krause [2006]; see also da Costa and Bueno [2009]).

(c) Existence. Quantification presupposes the existence of the objects that are quantified over; otherwise, certain inferences that are valid in classical logic end up being violated, such as: \(Fa \not\equiv \exists xFx\) (Lambert [2004]). On the usual interpretation of existential quantification, if \(a\) is \(F\), it follows that there exists an object that is \(F\) (\(a\) guarantees this to be the case). This highlights, however, an existential presupposition of the underlying inference. Arguably from the fact that Mickey Mouse is a talking mouse, we should not conclude that there exists an object that is a talking mouse (or that talking mice exist).

(d) Indeterminacy. Certain logical laws presuppose that there are no ‘gaps’, no indeterminacy, in reality, in the sense that it is definitely determined, for every \(p\), that \(p \lor \neg p\) (that is, excluded middle holds). Consider, however, a range of objects for which it is not determined whether \(p\) is the case or not. This may be due to the indeterminacy of some of their properties. (In some cases, vagueness can also raise this kind of issue, but it is important to note that vagueness and indeterminacy are very different phenomena; see, for instance, Greenough [2003], p. 245.) It seems that the very possibility of indeterminate objects is ruled out on purely logical grounds. These are, however, substantively metaphysical assumptions rather than logical ones.

(e) Predication. To make sense of predication, \(Fa\)—that \(a\) is \(F\)—one needs properties. Prima facie, this means that one needs to be committed to properties in virtue of logic alone. As an alternative, suppose we adopted an extensional understanding of predication, say, in terms of set theory. To state that \(a\) is \(F\) just is to state that \(a\) is a member of the set of things that are \(F\). But we would then be committed to sets.

The extensionalist approach faces some troubles, given that it identifies properties that are, in fact, distinct, such as having a heart and having a circulatory system. Despite the fact that the same objects satisfy these properties,\(^1\) the properties in question are clearly different (see Zalta [2010]).

A related issue, connected with predication, provides another source of metaphysical import for logic: it’s the idea that there is a significant, hierarchical distinction between objects and properties (or functions). After all, predication establishes a particularly close connection between a term and something of a higher type: a predicate. This connection is typically interpreted in terms of the relation between an object and a property, in which an object has the relevant property. Understood in these terms, predication seems to presuppose the hierarchical distinction between objects and properties.\(^2\)

(f) Inconsistency. In classical logic, from a contradiction everything follows, that is, as noted above, explosion holds: \(A \land \neg A \equiv B\), for every \(B\). The validity of explosion requires that no contradiction—i.e. no statement of the form \(A \land \neg A\)—be true. After all, if there were true contradictions, the premise of the inference would be true and its conclusion, for any false statement \(B\), would be false. Explosion would have then been violated.

\(^{1}\) Let’s suppose, for the sake of argument, that this is the case; if not, different examples can be easily generated.

\(^{2}\) Erich Reck raised this point in discussion.
Classical logic identifies the presence of contradictions with triviality—that is, with the fact that, due to explosion, everything follows from a contradiction. Although contradictions should indeed be avoided, is it a matter of logic or of metaphysics that no contradictions are (or could be) true? It seems that underlying the fact that in classical logic contradictions cannot be tolerated, there is a metaphysical principle to the effect that objects cannot have inconsistent properties. But, in this case, it is metaphysics rather than logic that has priority.

(g) Formality. It may be argued that formality is a requirement for logic (Sher [2013]). Logical notions are understood, on a Tarskian formulation, as being invariant under transformations (in particular, permutations) of the objects in the domain (Tarski [1986]). Another way in which this formal component can be spelled out is by establishing an isomorphism between the relevant domains, ignoring the particular objects that are related by the isomorphism. Such isomorphism then induces a class of equivalence among the objects in the domain. Once again, the particular nature of the objects that are thus related is irrelevant.

But transformations, permutations and isomorphism are all functions, and thus have metaphysical import: a particular mathematical (or, at least, formal) framework needs to be in place in order to use them. They can be formulated in set theory (in which case they are sets), in category theory (in which case they are morphisms), or in second-order logic (in which case they are relations). In all of these instances, particular commitments to a given ontology emerge.

It may be argued that these particular commitments would only be in place if one attempts to provide a specification of the particular transformations, permutations or isomorphisms in the context of particular logics. If the discussion is conducted at a slightly more abstract level, the argument goes, the need for such specification doesn’t arise and the commitments vanish.

3. THE ARGUMENTS ASSESSED

Are the arguments above compelling? A number of considerations can be used to resist them. Let me discuss them in turn.

(a) Resisting the argument from domains. According to this argument, domains of quantification typically are sets that range over absolutely everything that exists. Thus, given the exclusion of inconsistent and incomplete objects, classical logic can be preserved.

But do model-theoretic domains need to be sets? There is one sense in which they need not. The scope of quantifiers can be specified in a variety of ways, and some of them need not invoke sets. The use of sets in the model-theoretic view is just a convenience. As long as it is clear what objects are quantified over, this view can be developed and used. For instance, one can arguably replicate the content of set theory without invoking sets by reformulating it in second-order mereology with plural quantification (Lewis [1991]). In this case, as long as there are enough mereological atoms (in fact, inaccessibly many of them are required), the commitment to sets can be avoided.3

One may say that this suggestion simply provides a trade-off: instead of sets, we are committed to mereological atoms. What is thereby gained? Mereology’s commitments are arguably less metaphysically controversial than those found in set theory. Since the concern is

3 Of course, the concept of inaccessibly many is inherently set-theoretical. Thus, some more work is still needed to bypass set theory completely. But at least sets need not be invoked to characterize domains of interpretation directly.
with the metaphysical commitments of the model-theoretic conception, if that conception can be articulated with fewer commitments, one thereby obtains an advantage.

Alternatively, instead of a set of objects in the domain, all that is needed for quantification over certain objects are the objects. Plural quantification provides a clear way of formulating this point in the case of monadic second-order logic (Boolos [1998]). In order to quantify over runners, we don’t need a set (namely, the set of runners); we only need the runners. The same goes for any objects that are quantified over. Sets are just a convenient device in this context.

Someone may argue that there are different, non-equivalent, frameworks to characterize a domain of interpretation (set theory, mereology, etc.), and this suggests that it really doesn’t matter how the domain turns out to be formulated—any such formulation will do. But this is not right. For these formulations have different commitments and make different assumptions about how the domains are like. Thus, the resulting semantics, depending on the framework that is adopted, may turn out not to be equivalent, given differences in expressive resources available. For instance, if one of the frameworks is a set theory in which the axiom of choice holds, it is possible to prove the completeness theorem in a way that it isn’t if that axiom is not available. (I’ll return to this point below.) So, clearly, the particular framework does matter.

It may be thought that as long as logical principles and logical inferences are not interpreted, and are only taken as syntactic devices in a formal language, no metaphysical assumptions are involved. The principles and inferences in question can be formulated simply as transformation rules, with no assumption about their metaphysical import being made. An application of modus ponens in a particular formal system simply licenses the derivation of the consequent of a conditional given the relevant conditional and its antecedent. The rule itself doesn’t express anything beyond the fact that the particular substitution is permissible.

Of course, even this much already betrays a particular interpretation—admittedly a minimal one—according to which logical rules have a modal force: These rules allow certain inferences to be performed (that is, the inferences in question are permissible, given the rules of the system), and fail to ratify others (that is, other inferences are not permissible, since they are presumably subject to counterexamples). And, of course, we can only make sense of these rules because they are interpreted in a metalanguage (whatever metalanguage that is used in the particular case). Without at least some interpretation, we would not be able even to know exactly what is to follow the rules under consideration. Typically, to each formulation of object-level logical connectives, there is some connective in the metalanguage that corresponds (at least in part) to it, and this correspondence is crucial to make sense of the connective in the object language. Absence such correspondence, the meaning of the connective becomes highly underdetermined. (It may be underdetermined even with such correspondence, let alone without it!)

(b) Resisting the argument from identity. A concern can be raised about the intelligibility of a quantification that does not presuppose the identity of the objects that are quantified over. In the case of quantum particles, a procedure could have been devised to measure the cardinality of these particles, without presupposing that they have an ordinal. Domenech and Holik [2007] describe the potential process as follows:

For example, we could count how many electrons has an Helium atom imagining the following process (perhaps

---

4 To display a counterexample, one needs to provide a particular interpretation of the inference in question, since a case in which the premises of the rule are (clearly) true and the conclusion is (clearly) false needs to be exhibited. So, even at this basic level, a formal system cannot be purely formal.
not the best, but possible in principle). Put the atom in a cloud chamber and use radiation to ionize it. Then we would observe the tracks of both, an ion and an electron. It is obvious that the electron track represents a system of particle number equal to one and, of course, we cannot ask about the identity of the electron (for it has no identity at all), but the counting process does not depend on this query. The only thing that cares is that we are sure that the track is due to a single electron state, and for that purpose, the identity of the electron does not matter. If we ionize the atom again, we will see the track of a new ion (of charge $2e$), and a new electron track. Which electron is responsible of the second electron track? This query is ill defined, but we still do not care.

Now, the counting process has finished, for we cannot extract more electrons. The process finished in two steps, and so we say that an Helium atom has two electrons, and we know that, as the wave function of the electrons is an eigenstate of the particle number operator, no problem of consistence will arise in any other experiment if we make this assertion (Domenech and Holik [2007], p. 862).

It seems, however, that in this process the notion of identity is still presupposed. After all, when the atom is ionized for the second time, “we will see the track of a new ion (of charge $2e$), and a new electron track”. But if the track is of a new ion, and if it is a new electron track, it is presupposed that we are not extracting the same electron, otherwise we should count just one electron rather than two. As a result, the identity of the electrons that are quantified over seems to be presupposed in the end.

Identity is such a fundamental concept that it is very difficult to see how it could be avoided (see Bueno [2014] for further discussion). What this means, however, is that if the best case for the non-applicability of identity to quantum particles ultimately presupposes the identity of these objects, perhaps identity is indeed a logical notion after all. It is a concept that has the same (!) extension in every interpretation, as one would expect from a logical concept. Of course, if certain languages are impoverished enough not to be able to express identity, the concept is presupposed for the intelligibility of the relevant language. Even in classical propositional logic, in which identity is not normally expressed, $(A \lor \neg A)$ can only be a tautology if the same $A$ is exhibited in each disjunct. So identity is still presupposed. Finally, it is often claimed that identity can be defined in a second-order language, as follows:

$$x = y \text{ if, and only if, } \forall P (Px \leftrightarrow Py).$$

But, clearly, identity is presupposed even in this formulation, given that the variables occurring on the left side of the biconditional need to be the same semantic role as those occurring on the right side.

(c) Resisting the argument from existence. The existential quantifier need not be interpreted as being ontologically loaded (Azzouni [2004]). In fact, in the traditional Quinean formulation, two very different roles are assigned to the existential quantifier: on the one hand, it is used to specify that some objects in the domain are quantified over (rather than all such objects); on the other hand, it is also used to express the commitment to the existence of something. These are, however, very different roles (Azzouni [1997]). After all, we often quantify over objects we don’t think exist. For example, consider the statement: “Some fictional mice do not exist”. On the traditional understanding, this statement would be contradictory, since it would state that there exist fictional mice that do not exist. The statement, however, doesn’t seem to be contradictory. It just states that among fictional mice some do not exist (perhaps other do).

In order to keep these two roles of the existential quantifier apart, an existence predicate (‘$E$’) needs to be introduced in the underlying language. This predicate expresses the ontological commitments. Which conditions should such predicate satisfy? Clearly, this is not an issue that
should be settled by logic. What exists (or what fails to exist) is not a logical matter, but a metaphysical one. It is an issue to be settled on the basis of empirical, conceptual, and metaphysical considerations. This is as it should be. In particular, the statement above about fictional mice can be easily and consistently expressed as: $\exists x (Fx \land Ex)$, where ‘$F$’ stands for the predicate fictional mouse and ‘$E$’ stands for the existence predicate. Moreover, inferences from $Fa$ to $\exists x Fx$ can be preserved, without any commitment to nonexistent objects. After all, from the fact that $a$ is $F$, it follows that something is $F$, independently of the issue of $a$’s existence or not, for ontological import is assigned to the quantifiers. They are read as being ontologically neutral. Just as the existential import of the universal quantifier has been left behind as an unnecessary feature of traditional syllogisms, the existential important of the existential quantifier should be similarly resisted (Lambert [2004]). Metaphysical issues should be examined and settled (if they can be settled) via metaphysical considerations rather than by attempting to present them as though they could be resolved as a matter of logic alone.

It is worth noting that Carnap was very sensitive to the issue of the ontological import of existential quantifiers, and he was worried about the fact that a statement such as $\exists x x = x$, in which the existence of at least one object is stated, is logically true (Carnap [1937], p. 140). In a passage I quoted in the beginning of this paper, he highlighted:

> If logic is to be independent of empirical knowledge, then it must assume nothing concerning the existence of objects (Carnap [1937], p. 140; italics in the original).

Carnap then noted that identity statements such as the one just referred to presuppose the existence of objects. This, however, needs to be avoided:

> [i]f, in order to separate logic as sharply as possible from empirical science, we intend to exclude from the logical system any assumptions concerning the existence of objects, we must make certain alterations in the forms of language used […] (Carnap [1937], p. 140).

The alteration recommended by Carnap was to insist that existential sentences can only be obtained from universal ones if a proper name is involved, that is, in those cases in which the domain is indeed non-empty (Carnap [1937], p. 141). This indicates the measure to which Carnap was prepared to stand for the neutrality of logic. (Carnap’s move, however, does not ultimately work. The proper name in the inference in question may stand for a nonexistent object, such as Mickey Mouse. In this case, the existential sentence, assuming an ontologically loaded quantifier, would still come out false.)

He also noted that this issue does not emerge for the languages developed in Carnap [1937] (that is, so-called Languages I and II), for they are not name languages, but rather coordinate languages: “The expressions of the type 0 here designate not objects but positions” (Carnap [1937], p. 141). As a result, a statement such as $\exists x x = x$ expresses only

that at least one position exists. But whether or not there are objects to be found at these positions is not stated (Carnap [1937], p. 141).

One may wonder why the commitment to positions should be less troublesome for a neutralist view of logic than a commitment to objects. Presumably any existential assumptions from logic, of whatever kind, should be considered problematic. And to make sense of positions without objects seems to require some sort of coordinate system, and it is unclear why the existence of such a system should be true as a matter of logic alone.
Finally, Language II of Carnap [1937] is particularly strong, and it includes a formulation of the axiom of choice (which is called “Axiom of Selection”; see Carnap [1937], p. 92, axiom PSII 21; see also p. 142). Carnap is, of course, aware of the existential content of this axiom, which entails the existence of a selection class even if such a class cannot be defined; the non-constructive nature of the axiom is, of course, clear to him. Curiously, however, Carnap considers the issue of the assumption made by the axiom “as purely one of expedience” ([1937], p. 142). Given the clear existential content of the axiom, some strategy is needed here to justify its introduction and the corresponding avoidance of the resulting commitments. Simply gesturing toward the expedience of the axiom is not enough.

It seems that a better alternative is to invoke, once again, ontologically neutral quantifiers, which bypass altogether the issue of a possible commitment to positions or to the existential content of the axiom of choice.

(d) *Resisting the argument from indeterminacy.* Classical logic does make the presence of gaps due to indeterminacy (or vagueness) intolerable. But one can note that this is just a representational feature of classical logic rather than a substantive metaphysical commitment. As long one understands indeterminacy (and vagueness) as features of the language we use to describe the world, they need not be taken as features of the world. This blocks the need for logic to do metaphysical work, while still acknowledging that it is a controversial issue regarding the nature of indeterminacy (and vagueness) whether we are dealing with ontological or linguistic phenomena.⁵

There is an additional benefit of recognizing indeterminacy (and vagueness) as representational features of language: one need not assume by fiat that the relevant features, which seem to be so salient in natural language, exist or not as a matter of logic alone. The solutions to issues of indeterminacy (and vagueness) should not be obtained by fiat simply as a matter of logic. Whether objects have or lack certain properties is something that depends on the particular arrangements of the objects and their traits. It shouldn’t be something that is settled just by logic. Classical logic would force one to settle these issues independently of any additional considerations. Leaving these issues open, while their investigation is implemented by proper means—such as through the investigation of the metaphysics of indeterminacy—is a significant advantage. Metaphysical debates are then laid out explicitly in the open rather than dug into classical logic’s formalism.

I am not suggesting that to address the issues of indeterminacy (or vagueness) a simple change in logic will do the trick. This is far from the case. The point here is rather that these are complex philosophical issues, which are better addressed by examining in detail the relevant arguments rather than simply settle them (in part) by the sheer adoption of a given logic. Not that the simple acceptance of logic would be enough. What the logic does, in the particular case of classical logic, is to foreclose some possible solutions to what is being investigated. And this is best left open to properly philosophical inquiry.

There are, of course, solutions to the problem of indeterminacy (and vagueness) constructed within classical logic, but this logic, as noted, significantly constraints the resulting approaches: the range of indeterminacy (or vagueness) of properties is limited, since every object is taken to be entirely determined. One could insist that classical logic only constrains the use of language in an idealized way. Natural languages are different in that their predicates admit of

---

⁵ For a useful collection of classical papers on vagueness, see Keefe and Smith (eds.) [1999].
indeterminacy—a possibility that is, indeed, best left open, but which suggests that the underlying logic may not be classical in the end.

(e) Resisting the argument from predication. Predication can be understood without recourse to properties or sets. It is a metaphysical reading of logic that requires properties to be in the range of predicates. But why is such a reading required? A less metaphysically committed reading would characterize predication as an expression of features of our language rather than a commitment to traits of the world.

The interpretation of predicates in terms of properties is, of course, perfectly natural. But it is not metaphysically neutral. Properties are typically understood as abstract objects. If a proper understanding of logic requires a commitment to abstract objects, this is a significant commitment and one that, other things being equal, is better avoided.

Properties can be quantified over without commitment to their existence, as long as the relevant quantifiers are ontologically neutral. In this way, although the predicate ‘is blue’ stands for the property being blue, there is no need for the additional commitment to the existence of this property. Alternatively, even if non-neutral quantifiers are used, it is still possible to block the commitment to various such properties. The predicate ‘is blue’ stands for blue things rather than for the property of being blue. This familiar strategy consists in resisting the temptation to reify by moving up one level: from objects to their properties. Instead of doing that, one remains at the object level: ‘is blue’ is a predicate for blue things. But we don’t have here a general account of properties, since it’s still possible that different properties have the same extension.

(f) Resisting the argument from inconsistency. As we saw, given that explosion holds in classical logic, everything follows from a contradiction. The explosive nature of this logic—that is, the fact that explosion holds in it—can be thought of as just an expression of the language we use to describe the world. It need not be taken as a feature of the world itself. That is, it need not be considered as the rejection of true contradictions in reality, which involves minimally the denial that any statement of the form ‘A & ¬A’ is true.

Of course, explosion can be interpreted in a metaphysically robust way, as rejecting the existence of objects with inconsistent properties. But this reading, again, is not required, regardless of how independently plausible it may be. Logic can be thought of as a tool of inference, and for some purposes—such as to reason deductively in the absence of inconsistent information—it may be perfectly adequate to adopt an explosive logic, such as classical logic. However, if inconsistencies need to be managed until a consistent successor is found, a better strategy would be to explore a non-explosive logic, such as a paraconsistent one (for a survey, see da Costa, Krause, and Bueno [2007]). The use of such a logic in no way requires a commitment to the existence of true contradictions. As opposed to dialetheism, according to which there are such contradictions (Priest [2006]), the use of a paraconsistent logic can be made independently of any such metaphysical interpretation. Once again, the point is to resist the need to interpret logic ontologically.

Understood as an inferential procedure, paraconsistent logic allows one to draw a distinction between two concepts that are identified in classical logic: inconsistency and triviality. Since explosion holds in classical logic, the presence of a contradiction (a form of inconsistency) leads to the derivability of every sentence (that is, triviality obtains). Moreover, if every sentence is derivable (triviality), then in particular so is a contradiction (inconsistency). The result is that, according to classical logic, inconsistency and triviality go hand in hand. But if explosion is not
taken as generally valid, it is possible to draw a line between inconsistency and triviality. There may be inconsistent theories, which involve a contradiction of the form $A \land \neg A$, but which are non-trivial, since not everything follows from it, given the failure of explosion. Using a paraconsistent logic, it is possible, for example, to study properties of the Russell set (the set of all sets that are not-members of themselves). There is no such set in classical set theories (assuming they are consistent). But in a paraconsistent set theory, this set can be studied: it has certain properties and lacks others (for discussion and references, see da Costa, Krause, and Bueno [2007]). This provides an additional illustration of how logic can be used without certain strong metaphysical assumptions. This doesn’t mean, of course, that no assumptions are involved. They are. But what is significant is that the assumptions are kept in check, and critically assessed as needed rather than simply guiding the philosophical discussions blindly.

(g) Resisting the argument from formality. According to the formality argument, logic is ultimately formal, and that formality can be expressed, in part, in terms of the permutation of objects in the domain of interpretation (Tarski [1966] and Sher [1991]). This proposal can, in principle, be presented abstractly, without specifying the particular kind of permutation that is involved, and in this way one could avoid explicit commitments that emerge from the mathematical framework at hand.

In response, even if one adopted a more abstract description of the formality requirement, any implementation of it presupposes a framework that specifies the relevant permutations. Absent such a framework, the apparent lack of commitment is just an expression of the promissory nature of the proposal: no developed proposal would, in fact, be advanced; only a generic sketch would be provided. Had a fully developed account been offered, particular commitments, such as one of those just mentioned, would be invoked. Thus, it’s not the case that a more general description would bypass the relevant commitments. It would only seem to do so given that no particular view regarding the nature of the formality and how it can be specified would have been properly advanced.

The only way of avoiding the commitments from formality would be by developing a particular nominalization strategy for the relevant mathematical framework (for a critical examination of some such strategies, see Bueno [2013]). For example, as recommended on several occasions, suppose that quantifiers are interpreted as not being ontologically loaded. In this case, however the formality requirement ends up being formulated mathematically, via transformations, permutations or isomorphisms, the sheer fact that one is quantifying over mathematical objects is not enough to generate a commitment to their existence. In this way, particular commitments to abstract objects can be avoided, as long as a suitable nominalization strategy (one that avoids commitments to the relevant ontology) is implemented.

4. An Alternative Proposal

Let us take stock. As I argued above, it is possible to resist each of the arguments that have been considered. But, in each case, a particular maneuver is required. This suggests that, assuming the usual understanding of logical principles and logical inferences, they do have particular metaphysical presuppositions. In order to avoid them, alternative readings of the quantifiers or the relevant connectives need to be offered. Several strategies were considered along the way: (i) Quantifiers can be interpreted as being ontologically neutral; this avoids commitment to domains of interpretation, as well as to the existence of objects that one quantifies over or that are invoked in the mathematical framework required for the specification of the formality requirement. (ii)
Quantification could be interpreted as not requiring identity (although concerns were raised about the intelligibility of this move). (iii) Predication can be understood without commitment to properties. (iv) Indeterminacy and vagueness can be understood as features of the language one uses, and as a result violations of excluded middle are possible. (v) Inconsistency and triviality can be distinguished, thus allowing one to entertain the former without commitment to the latter (not excluding inconsistencies to begin with, but without embracing the existence of true contradictions either). If these readings are motivated, there are good grounds to resist the acceptance of certain metaphysical assumptions of certain logical principles (particularly from classical logic), and a metaphysically less committed alternative can be put forward. Let me indicate the central features of this alternative.

(a) *Logical pluralism*. First, the alternative involves a *logical pluralist view*, which acknowledges that there is a plurality of logics adequate to a certain domain—and, in some cases, more than one logic for the same domain (da Costa and Bueno [2001], Bueno [2002], and Bueno and Shalkowski [2009]). In this way, it is possible to acknowledge and make sense of the plurality of logics that have been developed and that do play a significant role in particular areas—despite the fact that classical logic is, of course, the one that is typically prevalent.

In order to provide a logic one needs to generate a particular relation of logical consequence. There are different ways of introducing such a relation, some better than others. This relation can be formulated in a number of different mathematical frameworks. For instance, it can be formulated in various set theories, such as: Zermelo-Fraenkel (Fraenkel, Bar-Hillel, and Levy [1973]), von Neumann-Bernays-Gödel (Gödel [1940]), Kelley-Morse (Kelley [1955]), or Quine’s New Foundations (Quine [1937/1961]). The characterization can also be implemented in category theory, in second-order mereology (Lewis [1991]), or in a modal-structural framework (Hellman [1989]). But these mathematical frameworks are significantly different, and their differences do matter to the content of the resulting account of logical consequence. For example, Quine’s New Foundations (NF) is incompatible with the axiom of choice (Specker [1953]). Consider now the completeness theorem for first-order logic: each consistent set of first-order sentences has a model. If the cardinality of such a model is properly specified, the axiom of choice is required to establish this theorem; in fact, it then becomes equivalent to the axiom of choice (Bell [2013]). Thus, the fact that this axiom is absent in NF changes significantly the content of the notion of logical consequence, given that the full completeness theorem is not available in this setting.

The dependence on the particular mathematical framework that is used provides an important limitation for the model-theoretic notion of logical consequence: it does not apply to set theory in general, such as to ZFC (Zermelo-Fraenkel set theory with the axiom of choice). For no set is the model of ZFC (there is no set of all sets in ZFC), and thus it is not possible to apply the model-theoretic framework to that theory. This is a major limitation to this characterization of logical consequence (see Field [1989]).

In contrast, the notion of logical consequence is better formulated modally: an argument is valid if, and only if, the conjunction of its premises and the negation of its conclusion is impossible. Depending on how the relevant possibilities are formulated, different logics emerge: consistent and complete structures generate classical logic; consistent and incomplete structures generate constructive logics; inconsistent and complete structures generate paraconsistent logics; and inconsistent and incomplete structures generate non-alethic logics (see Bueno and Shalkowski [2009], Beall and Restall [2006], and da Costa, Krause, and Bueno [2007]).
It may be argued that to introduce a primitive modal concept is just to do bad metaphysics. Modal concepts, the argument goes, are obscure and need to be explained rather than assumed as part of one’s stock of primitive notions. But is this really the case? First, the objection seems to assume a now outdated positivist outlook, with its underlying skepticism about modality. Second, it is unclear on what grounds modal concepts are deemed obscure. The primitive notion assumed here is one of logical possibility. Considerations of what is possible or not are perfectly clear, and especially so in simple, ordinary cases. It is this notion of logical possibility that is used to make intuitive judgments about what follows from what: simple cases in which the conjunction of the premises of an argument and the negation of its conclusion is impossible. (More complex cases are eventually developed together with the articulation of particular logical principles and inferences.) This notion of logical possibility is, of course, constrained: certain situations can go together; others cannot, and thus exclude one another. The crucial feature is that this notion is not formulated in terms of possible worlds, or other metaphysically problematic concepts. This provides an additional reason why skepticism about this notion is not warranted.

The view advanced here supports a form of logical pluralism not logical relativism or logical nihilism, and avoids logical monism. Some logics are appropriate for certain domains—and, given a domain, typically several such logics are adequate to it—but inadequate for other domains; that is, we have a version of logical pluralism. For example, in a domain involving inconsistencies, several paraconsistent logics would be adequate, but classical and intuitionist logics would not be (see da Costa, Krause, and Bueno [2007]). However, given the fact, noted above, that logical principles fail in some domain or another, no logic is ultimately appropriate for every domain; that is, logical monism ultimately fails. For instance, the distributivity law fails in quantum domains (for a discussion see, e.g., da Costa and Bueno [2001]); the law of identity arguably does not hold in some interpretations of quantum mechanics (French and Krause [2006]), and the rule of existential introduction fails in domains involving fictional objects (Lambert [2004]). It’s also not the case that every logic is appropriate for every domain; that is, logical relativism is similarly not the case. For example, classical logic is inadequate to accommodate constructive features of mathematical reasoning, whereas quantum logics are not particularly suited to accommodate inferences involving tensed statements in ordinary language. But since some logics are appropriate to some domains, logical nihilism—according to which no logic holds in any domain—also doesn’t go through either (see Bueno [2011], and Bueno and Shalkowski [2009]). Intuitionist logics, for instance, are appropriate for constructive domains of mathematics; quantum logics are adequate for quantum domains, and temporal logics for tensed ones. We thus have a form of logical pluralism that yields neither logical relativism nor nihilism about logic, while being incompatible with logical monism.

(b) Ontologically neutral quantifiers. Second, as noted above, the proposal advanced here favors an interpretation of quantifiers that takes them as not ontologically loaded (Azzouni [2004]). In this way, some neutrality over metaphysical issues can be maintained, and one need not simply assume that certain issues are settled as a matter of logic alone. Metaphysical issues will emerge, but they are then localized where they should be—for instance, in the scope of the existence predicate. This seems to be the right move: logic is not supposed to settle existence questions, after all.

The neutrality that is often associated with logic is not something easy to obtain. Logical principles do have their metaphysical import. The central point of the discussion so far is to indicate their sources, and make one aware of them, so that the relevant philosophical issues are
not just assumed as being settled as a matter of logic without further considerations. The proper approach is to examine the relevant philosophical issues on independent grounds—logic should only be a guide to proper inference rather than a source of doctrine to settle metaphysical disputes.

(c) Logics as inferential procedures. Finally, metaphysical commitments are also avoided by considering logics as a tool of inference rather than as substantive body of doctrines about the world. Again, the information about the world should be obtained from the relevant features of reality rather than from logic.

There are important connections between this proposal and Carnap’s views. Carnap clearly favored some form of neutrality of logic. This is expressed, in part, by his principle of tolerance. As he notes:

In logic, there are no morals. Everyone is at liberty to build up his own logic, i.e. his own form of language as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments (Carnap [1937], p. 52; italics in the original).

The strategy Carnap seems to suggest to implement the neutrality of logic is to adopt a formalist characterization of logic, thus resisting the temptation to add a philosophical gloss to logical principles and inferences. As a result, each logic is considered as just a formal system. In this respect, logic as a formal system can be thought of as analogous to pure mathematics, in which mathematical structures can be studied independently of any connections they may have to the empirical world, and in contrast with applied mathematics, in which those connections are central. Carnap was also, in principle, sympathetic toward logical pluralism. The different systems developed in Carnap [1937] may seem to illustrate this point. However, strictly speaking, they are different languages (different frameworks if you will) rather than different logics. The underlying logic of all such Carnapian languages is classical logic. At any rate, in order to implement any such neutrality, a formalist view of logic is ultimately adopted.

There are, however, important differences between the views suggested here and Carnap’s. I favor a modal characterization of logical consequence rather than a syntactic one (as noted, a primitive modal notion of consistency is needed in the background anyway). Moreover, even Carnap needs to provide an interpretation of logic, in order for it to be applied, and semantic considerations then have to be invoked. Hence, a strictly formalist view of logic is not enough to make sense of the various uses that logics are put to.

Carnap eventually realized that semantic considerations were needed, which eventually led him to introduce the well-known distinction between internal and external questions, as a way of providing legitimacy for an empiricist talk of semantic content (Carnap [1950]). But, as a nominalization strategy, the introduction of different frameworks within which one could explore virtually anything (from universals to properties, from possible worlds to fundamental metaphysical structures) does not succeed. The strategy is just too broad, and one ends up nominalizing the content of virtually anything, including the sort of metaphysical theorizing Carnap has always been skeptical of. (For a critical assessment of Carnap’s views on ontology, see Bueno [2016].)

5. Conclusion

The considerations above suggest that, although prima facie logics—and, in particular, classical logic—may seem to have substantive metaphysical commitments, there is a way of interpreting
them that avoids some of these commitments. As a result, a more neutral view of logic—that acknowledges logical pluralism, supports a neutral reading of quantifiers, and takes logics as tools of inferences—is advanced. The view will have its own presuppositions—deriving from the specific domains that each particular logic is applied to—but these can be identified and examined as needed.\footnote{Special thanks are due to Newton da Costa, with whom I have discussed these issues over many years, and who have raised several of the points discussed in this paper. My thanks also go to Jody Azzouni, Robert Burch, Craig Callender, James Cargile, Walter Carnielli, Itala D’Ottaviano, Robert Garcia, Theodore George, Brie Gertler, Michael Hand, Jeremy Heis, Paul Humphreys, Déci Krause, Chris Menzel, Trenton Merricks, Gregory Pappas, Erich Reck, Tomoya Sato, Scott Shalkowski, Gila Sher, and Clinton Tolley for very helpful discussions and feedback.}

**REFERENCES**


